

EQUATIONS INVOLVING RADICALS

Solve each of the following equations. Indicate ALL checks that will need to be completed and show ALL work that checks your solutions.

1. $\sqrt{3x+4} = 2$

3. $\sqrt{3x+4} = -6$

5. $x = 3\sqrt{x}$

7. $x = 2\sqrt{-x-1}$

9. $\sqrt{x^2 - x - 4} = x + 2$

11. $3 + \sqrt{3x+1} = x$

13. $\sqrt{3x+7} = 3x+5$

15. $\sqrt{4x} - x + 3 = 0$

17. $\sqrt{x+7} + 3 = \sqrt{x-4}$

19. $\sqrt{2x} = \sqrt{3x+12} - 2$

21. $\sqrt{x+2} = 1 - \sqrt{3x+7}$

23. $\sqrt{3x+4} - \sqrt{x+1} = 1$

25. $\sqrt{2\sqrt{7x+2}} = \sqrt{3x+2}$

27. $\sqrt{10+3\sqrt{x}} = \sqrt{x}$

29. $3 - \sqrt{x} = \sqrt{2\sqrt{x}-3}$

31. $\sqrt[3]{4x+3} = \sqrt[3]{2x-1}$

33. $(x^2 + 24x)^{\frac{1}{4}} = 3$

35. $(2x-1)^{\frac{2}{3}} = x^{\frac{1}{3}}$

37. $\sqrt[4]{7x+12} + 3 = 0$

39. $\sqrt[4]{5x^2-4} = x$

2. $\sqrt{9-8x} = 12$

4. $7 - \sqrt{4-5x} = 9$

6. $(x^2 - 16)^{\frac{1}{2}} = 9$

8. $\sqrt{15-2x} = x$

10. $\sqrt{3-x+x^2} = x-2$

12. $2 + \sqrt{12-2x} = x$

14. $\sqrt{4x+13} + 1 = 2x$

16. $\sqrt{2x} - x = -4$

18. $\sqrt{x+5} - 2 = \sqrt{x-1}$

20. $\sqrt{5x+1} - \sqrt{3x} = 1$

22. $\sqrt{2x-5} - 2 = \sqrt{x-2}$

24. $\sqrt{5x-1} + \sqrt{2-x} = \sqrt{8x+1}$

26. $\sqrt{x} + 2 = \sqrt{4+7\sqrt{x}}$

28. $\sqrt{3-2\sqrt{x}} = \sqrt{x}$

30. $x^{\frac{3}{2}} - 3x^{\frac{1}{2}} = 0$

32. $\sqrt[3]{2x} = \sqrt[3]{5x+2}$

34. $(3x^2 + 52x)^{\frac{1}{4}} = 4$

36. $(x-3)^{\frac{2}{5}} = (4x)^{\frac{1}{5}}$

38. $\sqrt[5]{7x+12} + 3 = 0$

40. $x(2+x)^{-\frac{1}{2}} + (2+x)^{\frac{1}{2}} = 0$

Equations Involving Radicals

$$\begin{aligned} \textcircled{1} \quad \sqrt{3x+4} &= 2 \\ 3x+4 &= 4 \\ 3x &= 0 \\ x &= 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \sqrt{9-8x} &= 12 \\ 9-8x &= 144 \\ -8x &= 135 \\ x &= -\frac{135}{8} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \sqrt{3x+4} &= -6 \\ \emptyset \quad -6 &\not\geq 0 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad 7 - \sqrt{4-5x} &= 9 \\ -\sqrt{4-5x} &= 2 \\ \sqrt{4-5x} &= -2 \quad -2 \not\geq 0 \\ \emptyset \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad x &= 3\sqrt{x} \quad x \geq 0 \\ x^2 &= 9x \\ x^2 - 9x &= 0 \\ x(x-9) &= 0 \\ x &= 0 \quad x = 9 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad (x^2-16)^{x^2} &= 9 \quad 9 > 0 \\ \sqrt{x^2-16} &= 9 \\ x^2-16 &= 81 \\ x^2-97 &= 0 \\ x^2 &= 97 \\ x &= \pm \sqrt{97} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad x &= 2\sqrt{-x-1} \quad x \geq 0 \\ x^2 &= 4(-x-1) \\ x^2 &= -4x-4 \\ x^2+4x+4 &= 0 \\ (x+2)^2 &= 0 \\ x+2 &= 0 \\ x &= -2 \quad -2 \not\geq 0 \\ \emptyset \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad \sqrt{15-2x} &= x \quad x \geq 0 \\ 15-2x &= x^2 \\ x^2+2x-15 &= 0 \\ (x-3)(x+5) &= 0 \\ x &= 3 \quad x \neq -5 \\ -5 &\not\geq 0 \end{aligned}$$

$$(9) \sqrt{x^2 - x - 4} = x + 2 \quad x + 2 \geq 0$$

$$x^2 - x - 4 = x^2 + 4x + 4 \quad x \geq -2$$

$$-5x = 8$$

$$x = -\frac{8}{5} \checkmark$$

$$(10) \sqrt{3 - x + x^2} = x - 2 \quad x - 2 \geq 0$$

$$3 - x + x^2 = x^2 - 4x + 4 \quad x \geq 2$$

$$3x = 1$$

$$x = \frac{1}{3} \quad \frac{1}{3} \not\geq 2$$

\emptyset

$$(11) 3 + \sqrt{3x + 1} = x$$

$$\sqrt{3x + 1} = x - 3$$

$$x - 3 \geq 0$$

$$x \geq 3$$

$$3x + 1 = x^2 - 6x + 9$$

$$0 = x^2 - 9x + 8$$

$$0 = (x - 1)(x - 8)$$

$$x = 1 \quad x = 8 \checkmark$$

$$1 \not\geq 3$$

Eg Involving Rad Continued

⑫ $2 + \sqrt{12 - 2x} = x$

$$\sqrt{12 - 2x} = x - 2 \quad x - 2 \geq 0$$

$$12 - 2x = x^2 - 4x + 4 \quad x \geq 2$$

$$0 = x^2 - 2x - 8$$

$$0 = (x - 4)(x + 2)$$

$$\checkmark (x - 4) \quad x = -2$$

$$-2 \not\geq 2$$

⑬ $\sqrt{3x+7} = 3x+5$

$$3x+5 \geq 0$$

$$3x+7 = 9x^2 + 30x + 25$$

$$3x \geq -5$$

$$0 = 9x^2 + 27x + 18$$

$$x \geq -\frac{5}{3}$$

$$0 = x^2 + 3x + 2$$

$$0 = (x + 2)(x + 1)$$

$$x = -2$$

$$\checkmark (x + 1)$$

$$-2 \geq -\frac{5}{3}$$

⑭ $\sqrt{4x+13} + 1 = 2x$

$$\sqrt{4x+13} = 2x - 1$$

$$2x - 1 \geq 0$$

$$4x+13 = 4x^2 - 4x + 1$$

$$2x \geq 1$$

$$0 = 4x^2 - 8x - 12$$

$$x \geq \frac{1}{2}$$

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1)$$

$$\checkmark (x - 3) \quad x = -1$$

$$-1 \not\geq \frac{1}{2}$$

$$(15) \sqrt{4x} - x + 3 = 0$$

$$\sqrt{4x} = x - 3 \quad x - 3 \geq 0$$

$$4x = x^2 - 6x + 9 \quad x \geq 3$$

$$0 = x^2 - 10x + 9$$

$$0 = (x - 9)(x - 1)$$

$$(x = 9) \quad x = 1$$

$$1 \not\geq 3$$

$$(16) \sqrt{2x} - x = -4$$

$$\sqrt{2x} = x - 4 \quad x - 4 \geq 0$$

$$2x = x^2 - 8x + 16 \quad x \geq 4$$

$$0 = x^2 - 10x + 16$$

$$0 = (x - 8)(x - 2)$$

$$(x = 8) \quad x = 2$$

$$2 \not\geq 4$$

$$(17) \sqrt{x+7} + 3 = \sqrt{x-4}$$

$$\sqrt{x+3} + 3 \geq 0$$

$$x+7 + 6\sqrt{x+7} + 9 = x-4$$

$$\sqrt{x+3} \geq -3$$

$$6\sqrt{x+7} = -20$$

$$-11 \not\geq 0$$

$$\emptyset$$

(18)

$$\sqrt{x+5} - 2 = \sqrt{x-1}$$

$$x+5 - 4\sqrt{x+5} + 4 = x-1$$

$$-4\sqrt{x+5} = -10$$

$$4\sqrt{x+5} = 10 \quad 10 > 0$$

$$16(x+5) = 100$$

$$16x + 80 = 100$$

$$16x = 20$$

$$x = \frac{20}{16}$$

$$x = \frac{5}{4} \quad \frac{5}{4} > 1$$

$$\sqrt{x+5} - 2 \geq 0$$

$$\sqrt{x+5} \geq 2$$

$$x+5 \geq 4$$

$$x \geq 1$$

(19)

$$\sqrt{2x} = \sqrt{3x+12} - 2$$

$$2x = 3x+12 - 4\sqrt{3x+12} + 4$$

$$4\sqrt{3x+12} = x+16$$

$$16(3x+12) = x^2 + 32x + 256$$

$$48x + 192 = x^2 + 32x + 256$$

$$0 = x^2 - 16x + 64$$

$$0 = (x-8)^2$$

$$x = 8$$

$$\sqrt{3x+12} - 2 \geq 0$$

$$\sqrt{3x+12} \geq 2$$

$$3x+12 \geq 4$$

$$3x \geq -8$$

$$x \geq -\frac{8}{3}$$

$$x+16 > 0$$

$$x > -16$$

144
48

$$(20) \sqrt{5x+1} - \sqrt{3x} = 1$$

$$\sqrt{5x+1} = \sqrt{3x} + 1$$

$$5x+1 = 3x + 2\sqrt{3x} + 1$$

$$2x = 2\sqrt{3x}$$

$$x = \sqrt{3x}$$

$$x^2 = 3x$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

$$x=0 \quad x=3$$

$$\{0, 3\}$$

$$\sqrt{3x} + 1 \geq 0$$

$$\sqrt{3x} \geq -1$$

$$2x \geq 0$$

$$x \geq 0$$

$$(21) \sqrt{x+2} = 1 - \sqrt{3x+7}$$

$$x+2 = 1 - 2\sqrt{3x+7} + 3x+7$$

$$2\sqrt{3x+7} = 2x+6$$

$$\sqrt{3x+7} = x+3$$

$$3x+7 = x^2 + 6x + 9$$

$$0 = x^2 + 3x + 2$$

$$0 = (x+1)(x+2)$$

$$x = -1 \quad x = -2$$

Check does not work

$$1 - \sqrt{3x+7} \geq 0$$

$$-\sqrt{3x+7} \geq -1$$

$$\sqrt{3x+7} \leq 1$$

$$3x+7 \leq 1$$

$$3x \leq -6$$

$$x \leq -2$$

$$x+3 \geq 0$$

$$x \geq -3$$

Eq. Inv. Radicals

(22)

$$\sqrt{2x-5} - 2 = \sqrt{x-2}$$

$$\sqrt{2x-5} = 2 + \sqrt{x-2}$$

$$2x-5 = 4 + 4\sqrt{x-2} + x-2$$

$$x-7 = 4\sqrt{x-2}$$

$$x^2 - 14x + 49 = 16(x-2)$$

$$x^2 - 14x + 49 = 16x - 32$$

$$x^2 - 30x + 81 = 0$$

$$(x-27)(x-3) = 0$$

$$x=27, x=3$$

$$2 + \sqrt{x-2} \geq 0$$

$$\sqrt{x-2} \geq -2$$

$$x-2 \geq 4 \quad x \geq 6$$

$$x-7 \geq 0$$

$$x \geq 7$$

(23)

$$\sqrt{3x+4} - \sqrt{x+1} = 1$$

$$\sqrt{3x+4} = \sqrt{x+1} + 1$$

$$3x+4 = x+1 + 2\sqrt{x+1} + 1$$

$$2x+2 = 2\sqrt{x+1}$$

$$x+1 = \sqrt{x+1}$$

$$x^2 + 2x + 1 = x + 1$$

$$x^2 + x = 0$$

$$x(x+1) = 0$$

$$x+1 \geq 0$$

$$x \geq -1$$

$$x=0, x=-1$$

Eg Inv. Rad.

(24)

$$\sqrt{5x-1} + \sqrt{2-x} = \sqrt{8x+1}$$

$$5x-1 + 2\sqrt{(5x-1)(2-x)} + 2-x = 8x+1$$

$$2\sqrt{(5x-1)(2-x)} = 4x$$

$$\sqrt{(5x-1)(2-x)} = 2x \quad x \geq 0$$

$$10x - 5x^2 - 2 + x = 4x^2$$

$$0 = 9x^2 - 11x + 2$$

$$0 = (9x-2)(x-1)$$

$$x = \frac{2}{9} \quad x = 1$$

(25)

$$\sqrt{2\sqrt{7x+2}} = \sqrt{3x+2}$$

$$2\sqrt{7x+2} = 3x+2$$

$$3x+2 \geq 0$$

$$4(7x+2) = 9x^2 + 12x + 4$$

$$3x \geq -2$$

$$28x + 8 = 9x^2 + 12x + 4$$

$$x \geq -\frac{2}{3}$$

$$0 = 9x^2 - 16x - 4$$

$$0 = (9x+2)(x-2)$$

$$x = -\frac{2}{9} \quad x = 2$$

$$(26) \sqrt{x} + 2 = \sqrt{4 + 7\sqrt{x}}$$

$$x + 4\sqrt{x} + 4 = 4 + 7\sqrt{x}$$

$$x = 3\sqrt{x} \quad x \geq 0$$

$$x^2 = 9x$$

$$x^2 - 9x = 0$$

$$x(x-9) = 0$$

$$x=0 \quad x=9 \quad \checkmark$$

$$(27) \sqrt{10+3\sqrt{x}} = \sqrt{x}$$

$$10+3\sqrt{x} = x$$

$$3\sqrt{x} = x-10$$

$$x-10 \geq 0$$

$$x \geq 10$$

$$9x = x^2 - 20x + 100$$

$$0 = x^2 - 29x + 100$$

$$0 = (x-25)(x-4)$$

$$x=25 \quad x=4$$

$$(28) \sqrt{3-2\sqrt{x}} = \sqrt{x}$$

$$3-2\sqrt{x} = x$$

$$-2\sqrt{x} = x-3$$

$$2\sqrt{x} = -x+3$$

$$-x+3 \geq 0$$

$$4x = x^2 - 6x + 9$$

$$x \leq 3$$

$$0 = x^2 - 10x + 9$$

$$0 = (x-9)(x-1)$$

$$x=9 \quad x=1$$

Eg. $\ln x$ Rad

$$(29) \quad 3 - \sqrt{x} = \sqrt{2\sqrt{x} - 3}$$

$$9 - 6\sqrt{x} + x = 2\sqrt{x} - 3$$

$$12 + x = 8\sqrt{x}$$

$$144 + 24x + x^2 = 64x$$

$$x^2 - 40x + 144 = 0$$

$$(x - 36)(x - 4) = 0$$

$$\cancel{x = 36} \quad x = 4$$

$$3 - \sqrt{x} \geq 0$$

$$12 + x \geq 0$$

$$-\sqrt{x} \geq -3$$

$$x \geq -12$$

$$\sqrt{x} \leq 3$$

$$x \leq 9$$

**
(30)

$$x^{3/2} - 3x^{1/2} = 0$$

$$\sqrt{x^3} - 3\sqrt{x} = 0$$

$$\sqrt{x^3} = 3\sqrt{x}$$

$$x^3 = 9x$$

$$x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x(x - 3)(x + 3) = 0$$

$$x = 0 \quad x = 3 \quad x = -3 \quad \checkmark$$

$$(31) \quad \sqrt[3]{4x+3} = \sqrt[3]{2x-1}$$

$$4x+3 = 2x-1$$

$$2x = -4$$

$$x = -2 \quad \checkmark$$

$$(32) \quad \sqrt[3]{2x} = \sqrt[3]{5x+2}$$

$$2x = 5x+2$$

$$-3x = 2$$

$$x = -\frac{2}{3} \quad \checkmark$$

Eg: Involving Rad.

$$\begin{aligned} (33) \quad (x^2 + 24x)^{\frac{1}{4}} &= 3 \\ \sqrt[4]{x^2 + 24x} &= 3 \\ x^2 + 24x &= 81 \\ x^2 + 24x - 81 &= 0 \\ (x + 27)(x - 3) &= 0 \\ x = -27 \quad x = 3 \end{aligned}$$

$$\begin{aligned} (34) \quad (3x^2 + 52x)^{\frac{1}{4}} &= 4 \\ \sqrt[4]{3x^2 + 52x} &= 4 \\ 3x^2 + 52x &= 256 \\ 3x^2 + 52x - 256 &= 0 \\ (3x + 64)(x - 4) &= 0 \\ x = -\frac{64}{3} \quad x = 4 \end{aligned}$$

$$\begin{aligned} (35) \quad (2x-1)^{\frac{2}{3}} &= x^{\frac{1}{3}} \\ \sqrt[3]{(2x-1)^2} &= \sqrt[3]{x} \\ (2x-1)^2 &= x \\ 4x^2 - 4x + 1 &= x \\ 4x^2 - 5x + 1 &= 0 \\ (4x-1)(x-1) &= 0 \\ x = \frac{1}{4} \quad x = 1 \end{aligned}$$

$$\begin{aligned} (36) \quad (x-3)^{\frac{2}{5}} &= 4x^{\frac{1}{5}} \\ \sqrt[5]{(x-3)^2} &= \sqrt[5]{4x} \\ x^2 - 6x + 9 &= 4x \\ x^2 - 10x + 9 &= 0 \\ (x-9)(x-1) &= 0 \\ x = 9 \quad x = 1 \end{aligned}$$

$$\begin{aligned} (37) \quad \sqrt[4]{7x+12} + 3 &= 0 \\ \sqrt[4]{7x+12} &= -3 \\ \phi \end{aligned}$$

$$\begin{aligned} (38) \quad \sqrt[5]{7x+12} + 3 &= 0 \\ \sqrt[5]{7x+12} &= -3 \\ 7x+12 &= -243 \\ 7x &= -255 \\ x &= -\frac{255}{7} \end{aligned}$$

$$\begin{aligned} (39) \quad \sqrt[4]{5x^2-4} &= x \quad x \geq 0 \\ 5x^2 - 4 &= x^4 \\ 0 &= x^4 - 5x^2 + 4 \\ 0 &= (x^2 - 1)(x^2 - 4) \\ x^2 &= 1 \quad x^2 = 4 \\ x &= \pm 1 \quad x = \pm 2 \\ \{1, 2\} \end{aligned}$$

$$\begin{aligned} (40) \quad x(2+x)^{-\frac{1}{2}} + (2+x)^{\frac{1}{2}} &= 0 \\ \sqrt{2+x} \left(\frac{x}{\sqrt{2+x}} + \sqrt{2+x} \right) &= 0 \\ x + 2 + x &= 0 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$